TECHNICAL NOTES

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No. 866

A STUDY OF GENERAL INSTABILITY OF BOX BEAMS WITH TRUSS-TYPE RIBS

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E R R A T A

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RESTRICTED

Page 9, equation (14): The subscript 1 has been omitted from the first f. The equation should read:

$$\Delta h_t = \left[k_{t+1} f_1 (y_{t+1}) - \ldots \right]$$

Page 12, equations (28) and (29): g_1 and g_2 should not be written as subscripts. The equations should appear as follows:

$$H_1 = \frac{2b^2}{\pi^4 D_{x_1} g_1}$$

$$H_2 = \frac{2b^2}{\pi^4 D_{x_2} g_2}$$

Page 15, first paragraph under the equation: The following sentence should precede the last sentence in the paragraph:

"This assumption greatly simplifies the calculation."

Page 18, equation (53): The lower limit of the summations should be written "k = 0" in both places.

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SUMMARY

The design of truss-type ribs for box beams is theoretically treated with regard to the function of the ribs in stabilizing the compression flange. The theory is applied to a design problem, and the results of this application are presented and discussed in relation to the general problem of rib design. The results of some tests made as a part of this general study are presented in an appendix.

INTRODUCTION

When airplane wings were constructed with fabric covering, the main function of the ribs was to support the fabric and to transmit the air loads to the spars. All the bending strength of the wing lay in these spars. With the adoption of the stressed-skin type of structure, designers naturally followed principles established in the previous type of construction. Thus, some of the first stressed-skin wings were mainly a reproduction of the previous type of construction with the fabric replaced by metal skin. This tendency still remains. Even today, after the flange material of the spars has been spread out over the upper and the lower surfaces of the wing to form the covering, the ribs inside are still proportioned on the basis of a nonstressed fabric covering.

In the design of a stressed-skin wing, the bending strength of the wing is commonly assumed to depend upon the compressive strength of the panels between ribs. The end fixity of these panels is also assumed, in many cases, to be greater than unity. These assumptions are sound provided that the ribs are properly designed.

Theoretical studies indicate that, of all the functions of a rib in a stressed-skin wing, the most important is to stabilize the distributed flange material on the compression side of the wing. If the rib has sufficient stiffness, it will stabilize the compression flange so thoroughly that a buckling failure can occur only between ribs. If the rib has less than this required stiffness, however, the buckle pattern will not be confined between ribs but will involve displacements of the ribs and possible failure of one or more ribs. This second type of buckling has been referred to as a "general instability" of the wing structure.

The results of a preliminary study of this general instability problem are presented in reference 1, where only a compression flange was assumed to be present. The presence of the tension flange is an important factor in stabilizing the compression flange through the intermediary of the ribs. In this paper, the effect of the tension flange has been included as well as the effect of other factors.

The theory presented herein was developed in connection with the planning of a research program on the general instability of stressed-skin wings. The results of some tests made as a part of this program are presented in an appendix.

ASSUMPTIONS

It is assumed ~~that~~: the tension and the compression flanges are flat rectangular plates; the box beam is symmetrical and subjected to pure bending in the plane of symmetry; all cross sections have the same dimensions; the bending stresses are uniformly distributed over the flanges; the ribs are similar, equally spaced, and have zero torsional rigidity; and the material is elastic. With these assumptions, the flanges may be regarded as flat plates under edge load with continuous support along the shear webs and the end bulkheads and with elastic support along each rib. All displacements are assumed to be small and normal to the flanges.

SYMBOLS

Where any of the following symbols appear in the report with subscripts 1 and 2, these subscripts refer to the tension (lower) and the compression (upper) flange, respectively.

x	coordinate axis in spanwise direction
y	coordinate axis in chordwise direction
z	coordinate axis normal to the span-chord plane
I_x, I_y	moment of inertia per inch of flange for bending in x and y directions
E	tension-compression modulus ($E = 10^7$ lb per sq in. for 24S-T aluminum alloy)
G	shear modulus $[G = E/2(1 + \mu) = 0.385E]$
GJ	torsional rigidity per inch of flange
J	torsional stiffness constant, per inch of flange
B	flexural rigidity EI of rib chord
μ	Poisson's ratio for the material ($\mu = 0.3$ for 24S-T aluminum alloy)
μ_x, μ_y	Poisson's ratio for load applied in x and y directions
a	length of flange
b	width of flange between shear webs when used in design of rib to span distance between shear webs; width of flange between panel points of rib truss when used in design of rib chord to span distance between panel points
P	total load on flange of width b
v	displacement in y direction, positive to right, used only to designate displacement of panel points of rib truss

w	displacement in z direction, positive upward
$f(y)$	a nondimensional function of y giving relative values of w in sections parallel to y axis
$C_1, C_2, \ldots C_i$	distance of ribs from y axis
L	spacing of ribs
m	integer with values 1, 2, 3, etc.
j	integer giving number of rib spaces in length of plate $(j = a/L)$
k	integer with values 0, 1, 2, 3, etc.
λ	half wave length of buckle
$A_{r,s,t}$	area of web or chord member of rib
$h_{r,s,t}$	length of web or chord member of rib
$y_{r,s,t}$	y-coordinate of end of web or chord member of rib
k_t	ratio of displacements v/w, for panel point containing left end of tth lower chord member of rib
k_s	ratio of displacements, v/w for panel point containing left end of sth upper chord member of rib
k_{1_r}	ratio of displacements v/w for panel points containing lower end of rth web member of rib
k_{2_r}	ratio of displacements v/w for panel point containing upper end upper end of rth web member of rib
α_r	angle with vertical made by rth web member of rib, positive if web member slopes upward toward the right
Q	ratio of area to length of first web member of rib
D_x, D_y	flexural rigidity per inch of flange in x and y directions

$$D_x = \frac{(EI)_x}{1 - \mu_x\mu_y} \qquad D_y = \frac{(EI)_y}{1 - \mu_x\mu_y}$$

$$\beta = a/b \qquad r = Pb/\pi^2 D_x \qquad s = gD_y/D_x \qquad \gamma = B/bD_x$$

$$l = (2/b) \int_0^b [f(y)]^2 \, dy$$

$$g = (2b^3/\pi^4) \int_0^b [\partial^2 f/\partial y^2]^2 \, dy$$

$$t = \frac{4}{\pi^2} \frac{GJb}{D_x} \int_0^b \left(\frac{\partial f}{\partial y}\right)^2 dy - \frac{2}{\pi^2} b \left(\mu_y + \frac{D_y}{D_x}\mu_x\right) \int_0^b \frac{\partial^2 f}{\partial y^2} f(y) \, dy$$

THE GENERAL-INSTABILITY PROBLEM

The energy method of solving buckling problems as described by Timoshenko (reference 2) avoids certain mathematical difficulties and is therefore used herein. In reference 1, where this method was also used, it was found that the final equation for general instability involving the ribs could not be solved for the critical buckling load. For an assumed critical load, however, it was observed that this final equation could be readily solved for the rib stiffness. The same condition holds true for the equations developed in this paper.

When the assumed critical load is less than the panel strength for buckling between ribs, the final equation gives the rib size required if general instability is to occur at the assumed critical load. The allowable panel strength is the highest critical load that can possibly be developed. Consequently, in the limit as the assumed critical load approaches the allowable panel strength, the formula for general instability gives the rib size required to make valid the basic assumption commonly made by the designer; namely, that the bending strength of the wing depends upon the panel strength between ribs.

From the foregoing discussion, it is evident that the

problem of general instability should have been studied as soon as stressed-skin structures were adopted. In most cases the ribs provided have, fortunately, been stiff enough to develop the panel strength between ribs. Only in recent designs, where visible evidence of the general instability failure was established, has the importance of the problem been generally recognized.

General-instability failure has been observed to occur in several stressed-skin wings with truss-type ribs. The theory of this report is limited to a consideration of symmetrical rectangular box beams with this type of rib. If desirable, the theory may be applied as an approximation to webbed ribs either with or without lightening holes.

The main purpose of a rib in a stressed-skin wing is to stabilize the compression flange. In the performance of this function, the rib acts as a beam or a truss spanning the distance between shear webs. If the rib is attached to the tension flange, it is aided by that flange in stabilizing the compression flange. The magnitude of this aid depends upon the relative dimensions of the entire structure and upon the loads in the flanges.

If the rib is of the truss type, the chords of the rib must support the flanges between the panel points of the rib truss. In the performance of this function the chord acts as a beam. If the rib chords are rigidly joined to the web members at the panel points, these beams have end restraint at the panel points. The magnitude of this restraint depends upon the relative dimensions of the rib chord and the web members. The end restraint has an important effect on the required stiffness of the rib chord, and this effect is computed from the theory of reference 1 as extended in this paper.

DESIGN OF RIB TO SPAN DISTANCE BETWEEN SHEAR WEBS

Theory.- The problem considered is that of two flat plates held apart by rigid supports in the form of shear webs along the side edges, rigid supports in the form of heavy ribs along the end edges, and elastic supports in the form of transverse ribs at equal intervals between the rigid end ribs. The loading is a uniform compression in the x-direction for the upper plate and a uniform tension

in the x-direction for the lower plate. (See fig. 1.) As stated previously, the solution herein presented is made on the basis of the energy method for the solution of buckling problems as used by Timoshenko in reference 2 (p. 378) and as applied in reference 1.

In the energy method, it is necessary to assume deflection equations consistent with the boundary conditions of the problem for both the tension and the compression flanges. These equations are, respectively,

$$w_1 = f_1(y) \sum_{m=1}^{m=\infty} a_m \sin \frac{m\pi x}{a} \tag{1}$$

$$w_2 = f_2(y) \sum_{m=1}^{m=\infty} b_m \sin \frac{m\pi x}{a} \tag{2}$$

The rib chords are assumed to follow these displacements.

The strain energy in a plate when buckling occurs is assumed to be given by the following equation:

$$V = \frac{1}{2} \int_0^b \int_0^a \left\{ D_x \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \mu_y \frac{\partial^2 w}{\partial y^2} \frac{\partial^2 w}{\partial x^2} \right] + D_y \left[\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \mu_x \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] + 2GJ \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right\} dxdy \tag{3}$$

On substitution of the deflection equations, the sum of the plate energies becomes

$$V_1 + V_2 = \frac{\pi^4}{8} \left\{ \frac{b}{a^3} \sum_{m=1}^{m=\infty} m^4 \left(D_{x_1} l_1 a_m{}^2 + D_{x_2} l_2 b_m{}^2\right) + \frac{a}{b^3} \sum_{m=1}^{m=\infty} \left(D_{y_1} g_1 a_m{}^2 + D_{y_2} g_2 b_m{}^2\right) + \frac{1}{ab} \sum_{m=1}^{m=\infty} m^2 \left(D_{x_1} t_1 a_m{}^2 + D_{x_2} t_2 b_m{}^2\right) \right\} \tag{4}$$

The bending energy of a prismatic bar is given by

$$V = \frac{B}{2}\int_0^b \left(\frac{\partial^2 w}{\partial y^2}\right)^2 dy \tag{5}$$

Hence the energy of bending of all the rib chords is given by

$$V_3 + V_4 = \frac{\pi^4}{4b^3}\left[g_1 B_1 \sum_1 \left(\sum_{m=1}^{m=\infty} a_m \sin\frac{m\pi C_1}{a}\right)^2 \right.$$
$$\left. + g_2 B_2 \sum_1 \left(\sum_{m=1}^{m=\infty} b_m \sin\frac{m\pi C_1}{a}\right)^2\right] \tag{6}$$

In accordance with the assumptions of truss action, the web members have only energy of axial displacement. For a prismatic bar, this energy is

$$V = \frac{1}{2}\frac{AE}{h}(\Delta h)^2 \tag{7}$$

Although in the case of the flanges only normal displacements are being considered, as is usual in the theory of small deflections, the fact that some of the panel points of the ribs must undergo chordwise displacement in order to develop a truss action will be taken into account. Thus, the elongation of a web member is

$$\Delta h = (w_2 - w_1)\cos\alpha + (v_2 - v_1)\sin\alpha \tag{8}$$

The angle α is considered positive if the web member slopes upward toward the right. (See fig. 2.) Substitution of the equations for w_1 and w_2 (equations (1) and (2) gives for the rth web member:

$$\Delta h_r = (\cos\alpha_r + k_{2_r}\sin\alpha_r)\, f_2(y_r) \sum_{m=1}^{m=\infty} b_m \sin\frac{m\pi C_1}{a}$$
$$- (\cos\alpha_r + k_{1_r}\sin\alpha_r)\, f_1(y_r) \sum_{m=1}^{m=\infty} a_m \sin\frac{m\pi C_1}{a} \tag{9}$$

where y_r denotes the y-coordinate of the end of the rth web member. The constants $k_{2_r} = \frac{v_{2_r}}{w_{2_r}}$ and $k_{1_r} = \frac{v_{1_r}}{w_{1_r}}$ de-

pend only on the geometry of the box beam. The energy in all the web members in all ribs is therefore

$$V_5 = \frac{E}{2} \sum_i \sum_r \frac{A_r}{h_r} (\Delta h_r)^2 \tag{10}$$

The elongation of an upper chord member of the rib truss with ends at the sth and the (s + 1)th panel points is

$$\Delta h_s = v_{s+1} - v_s = k_{s+1} w_{s+1} - k_s w_s$$

$$= \left[k_{s+1} f_2(y_{s+1}) - k_s f_2(y_s)\right] \sum_{m=1}^{m=\infty} b_m \sin \frac{m\pi C_i}{a} \tag{11}$$

where y_s and y_{s+1} are the y-coordinates of the ends of the sth member. On the assumption of a constant stress along the chord member, the energy in all upper chord members of all ribs is

$$V_6 = \frac{E}{2} \sum_i \sum_s \frac{A_s}{h_s} (\Delta h_s)^2 \tag{12}$$

Similarly, the energy in all lower chord members of all ribs is

$$V_7 = \frac{E}{2} \sum_i \sum_t \frac{A_t}{h_t} (\Delta h_t)^2 \tag{13}$$

where

$$\Delta h_t = \left[k_{t+1} f_1(y_{t+1}) - k_t f_1(y_t)\right] \sum_{m=1}^{m=\infty} a_m \sin \frac{m\pi C_i}{a} \tag{14}$$

The loss of potential energy T by the forces applied at the ends of either flange is given by the equation

$$T = \frac{P}{2b} \int_0^b \int_0^a \left(\frac{\partial w}{\partial x}\right)^2 dx\, dy \tag{15}$$

On substitution of the deflection equations (equations (1) and (2)), the total loss of potential energy of the tension and the compression flange forces is

$$T_1 + T_2 = \frac{\pi^2}{8a}\left(P_2 l_2 \sum_{m=1}^{m=\infty} m^2 b_m{}^2 - P_1 l_1 \sum_{m=1}^{m=\infty} m^2 a_m{}^2\right) \qquad (16)$$

The general equation for the calculation of the critical value of P_2 is obtained by equating the total strain energy V to the loss of potential energy T of the loads P_1 and P_2, or

$$T_1 + T_2 = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 \qquad (17)$$

On substitution of the values of the T's and the V's in equation (17), there is obtained, on solving for P_2, the following relation:

$$P_2 = \frac{\left[\begin{aligned}&\frac{E}{2}\sum_i\sum_r \frac{A_r}{h_r}(\Delta h_r)^2 + \frac{E}{2}\sum_i\sum_s \frac{A_s}{h_s}(\Delta h_s)^2 + \frac{E}{2}\sum_i\sum_t \frac{A_t}{h_t}(\Delta h_t)^2 \\ &+ \frac{\pi^4}{8}\left\{\frac{b}{a^3}\sum_{m=1}^{m=\infty} m^4 (D_{x_1} l_1 a_m{}^2 + D_{x_2} l_2 b_m{}^2) + \frac{a}{b^3}\sum_{m=1}^{m=\infty} (D_{y_1} g_1 a_m{}^2 + D_{y_2} g_2 b_m{}^2)\right. \\ &\left. + \frac{1}{ab}\sum_{m=1}^{m=\infty} m^2 (D_{x_1} t_1 a_m{}^2 + D_{x_2} t_2 b_m{}^2)\right\} + \frac{\pi^2}{8a} P_1 l_1 \sum_{m=1}^{m=\infty} m^2 a_m{}^2 \\ &+ \frac{\pi^4}{4b^3}\left\{g_1 B_1 \sum_i \left[\sum_{m=1}^{m=\infty} a_m \sin\frac{m\pi C_i}{a}\right]^2 + g_2 B_2 \sum_i \left[\sum_{m=1}^{m=\infty} b_m \sin\frac{m\pi C_i}{a}\right]^2\right\}\end{aligned}\right]}{\frac{\pi^2}{8a}\, l_2 \sum_{m=1}^{m=\infty} m^2 b_m{}^2} \qquad (18)$$

The coefficients a_m and b_m must be so chosen as to give a minimum P_2. This condition is obtained by setting the derivative of P_2 with respect to each coefficient equal to zero. Thus a system of homogeneous linear equations in a_1, a_2, a_3, ... b_1, b_2, b_3, ... is obtained. The solution of these equations yields values of a_1, a_2, a_3, ... b_1, b_2, b_3, ... different from zero only if the determinant formed by the coefficients of a_1, a_2, a_3, ... b_1, b_2, b_3, ... is equal to zero. This determinant can be factored. When each factor in turn is set equal to zero, an equation is obtained relating the critical loads

to the characteristics of the ribs and the plates for the particular wave pattern described by that factor. If the factors that describe wave patterns with a node at each rib are each set equal to zero, the rib characteristics do not enter the equation obtained. If the factors that describe all other wave patterns are each set equal to zero, it is found that all the resulting equations can be represented after suitable arrangement for practical calculations by the following single relation:

$$Q = \frac{-b' \pm \sqrt{(b')^2 - 4a'}}{2a'} \tag{19}$$

where

$$a' = \left(\frac{j}{2}\right)^2 (ed-fc) \left[\sum_{k=o}^{k=\infty} \frac{1}{J_{2jk+q}} + \sum_{k=o}^{k=\infty} \frac{1}{J_{2j(k+1)-q}}\right] \left[\sum_{k=o}^{k=\infty} \frac{1}{R_{2jk+q}} + \sum_{k=o}^{k=\infty} \frac{1}{R_{2j(k+1)-q}}\right] \tag{20}$$

$$b' = \frac{jf}{2}\left[\sum_{k=o}^{k=\infty} \frac{1}{J_{2jk+q}} + \sum_{k=o}^{k=\infty} \frac{1}{J_{2j(k+1)-q}}\right] - \frac{jc}{2}\left[\sum_{k=o}^{k=\infty} \frac{1}{R_{2jk+q}} + \sum_{k=o}^{k=\infty} \frac{1}{R_{2j(k+1)-q}}\right] \tag{21}$$

$q = 1, 2, 3, \ldots (j - 1)$

$$J_m = \frac{(l_1 r_1 + t_1)\beta^2 m^2 + l_1 m^4 + s_1 \beta^4}{2g_1 \beta^3} \tag{22}$$

$$R_m = \frac{(l_2 r_2 - t_2)\beta^2 m^2 - l_2 m^4 - s_2 \beta^4}{2g_2 \beta^3} \tag{23}$$

$m = 2jk+q \quad \text{or} \quad 2j(k+1) -q$

$$e = H_1 E \sum_r W_r(\cos \alpha_r + k_{2_r} \sin \alpha_r)(\cos \alpha_r + k_{1_r} \sin \alpha_r) f_2(y_r) f_1(y_r) \tag{24}$$

$$f = H_1 E \left\{ \sum_r W_r \left[(\cos \alpha_r + k_{1_r} \sin \alpha_r)\, f_1(y_r) \right]^2 + \sum_t W_t \left[k_{t+1} f_1(y_{t+1}) - k_t f_1(y_t) \right]^2 \right\} + \frac{B_1}{QbD_{x_1}} \tag{25}$$

$$c = H_2 E \left\{ \sum_r W_r \left[(\cos \alpha_r + k_{2_r} \sin \alpha_r)\, f_2(y_r) \right]^2 + \sum_s W_s \left[k_{s+1} f_2(y_{s+1}) - k_s f_2(y_s) \right]^2 \right\} + \frac{B_2}{QbD_{x_2}} \tag{26}$$

$$d = H_2 E \sum_r W_r (\cos \alpha_r + k_{2_r} \sin \alpha_r)(\cos \alpha_r + k_{1_r} \sin \alpha_r) f_2(y_r) f_1(y_r)$$
$$= \frac{H_2}{H_1} e \tag{27}$$

$$H_1 = \frac{2b^2}{\pi^4 D_{x_1} g_1 \xi_1} \tag{28}$$

$$H_2 = \frac{2b^2}{\pi^4 D_{x_2} g_2 \xi_2} \tag{29}$$

$Q = \dfrac{A_1}{h_1}$, where A_1 and h_1 refer to the first diagonal (30)

$W_r = \dfrac{A_r/h_r}{A_1/h_1}$, where A_r and h_r refer to the rth diagonal (31)

$W_t = \dfrac{A_t/h_t}{A_1/h_1}$, where A_t and h_t refer to the tth bottom chord member of rib (32)

$W_s = \dfrac{A_s/h_s}{A_1/h_1}$, where A_s and h_s refer to the sth top chord member of rib (33)

The last term, which contains Q, in each of equations (25) and (26) is small in relation to the other terms in these equations. Consequently, these terms containing Q may be neglected and equation (19) will give Q explicitly. The sign in front of the radical in equation (19) should be so chosen as to give Q a maximum positive value. Now j is the number of spaces into

which the box beam is divided by the ribs. The various wave patterns are described by integral values of q where $q = 1, 2, 3$, etc., but q does not exceed $(j - 1)$. Hence there are $(j - 1)$ wave patterns represented by equation (19). In any problem the value of q should be so chosen as to give a maximum positive value for Q. After the establishment of Q, the required sizes of all rib members are obtained from the necessarily assumed ratios W_r, W_t, and W_s.

Equation (19) applies to a box beam of finite length a, where $a = jL$. In this beam the side and the end edges of each flange are assumed to be rigidly supported. The effect of the rigid end supports diminishes as the length of the beam increases. In the limit as a, and hence j, approaches infinity, equation (19) becomes

$$Q = \frac{-b' \pm \sqrt{(b')^2 - 4a'}}{2a'} \tag{34}$$

where

$$a' = g_1 g_2 \left(\frac{L}{b}\right)^6 (ed - fc) \left(\sum_{k=o}^{k=\infty} \frac{1}{J_k{}'} + \sum_{k=o}^{k=\infty} \frac{1}{J_k{}''}\right) \left(\sum_{k=o}^{k=\infty} \frac{1}{R_k{}'} + \sum_{k=o}^{k=\infty} \frac{1}{R_k{}''}\right) \tag{35}$$

$$b' = g_1 \left(\frac{L}{b}\right)^3 f \left(\sum_{k=o}^{k=\infty} \frac{1}{J_k{}'} + \sum_{k=o}^{k=\infty} \frac{1}{J_k{}''}\right) - g_2 \left(\frac{L}{b}\right)^3 c \left(\sum_{k=o}^{k=\infty} \frac{1}{R_k{}'} + \sum_{k=o}^{k=\infty} \frac{1}{R_k{}''}\right) \tag{36}$$

where

$$J_k{}' = (l_1 r_1 + t_1) \left(\frac{L}{b}\right)^2 \left(2k + \frac{L}{\lambda}\right)^2 + s_1 \left(\frac{L}{b}\right)^4 + l_1 \left(2k + \frac{L}{\lambda}\right)^4 \tag{37}$$

$$J_k{}'' = (l_1 r_1 + t_1) \left(\frac{L}{b}\right)^2 \left(2k+2 - \frac{L}{\lambda}\right)^2 + s_1 \left(\frac{L}{b}\right)^4 + l_1 \left(2k+2 - \frac{L}{\lambda}\right)^4 \tag{38}$$

$$R_k' = (l_2 r_2 - t_2)\left(\frac{L}{b}\right)^2 \left(2k + \frac{L}{\lambda}\right)^2 - s_2 \left(\frac{L}{b}\right)^4 - l_2 \left(2k + \frac{L}{\lambda}\right)^4 \qquad (39)$$

$$R_k'' = (l_2 r_2 - t_2)\left(\frac{L}{b}\right)^2 \left(2k+2 - \frac{L}{\lambda}\right)^2 - s_2 \left(\frac{L}{b}\right)^4 - l_2 \left(2k+2 + \frac{L}{\lambda}\right)^4 \qquad (40)$$

In a box beam of finite length, any one of $(j - 1)$ wave patterns is possible. In the box beam of infinite length, any one of an infinite number of wave patterns is possible. The particular wave pattern, determined by λ, that gives the highest value of Q is the one which must be used in the design.

Design problem.- The detailed application of the preceding theory to a practical design problem follows the general outline of the example given in reference 1. Consequently, only the results of the application will be given and briefly discussed, insofar as they bear upon the practical design of box beams with flat flanges.

The dimensions of the ribs and the flanges are given in figures 2 and 3. The material is 24S-T aluminum alloy. The short-column and the long-column curves for this flange material are assumed to be, respectively, for pin ends,

$$\frac{P}{A} = 48{,}500 - \frac{(48500)^2 \left(\frac{L}{\rho}\right)^2}{4\pi^2 \; 10^7} \qquad (41)$$

and

$$\frac{P}{A} = \frac{\pi^2 \; 10^7}{\left(\frac{L}{\rho}\right)^2} \qquad (42)$$

The box beam is assumed to have concentrations of material in the four corners in such amounts that, under the design bending moment, the forces in the 76-inch-wide flanges are as follows:

Tension flange . . . . 96,330 pounds

Compression flange . . . . 177,670 pounds

The compression-flange force of 177,670 pounds is the panel strength between ribs for pin ends. The problem is to apply equation (19) and to find the value of Q required to cause buckling at these assumed flange loads.

The flanges of the box beam, being flat plates, are very rigid in their own plane. When buckling occurs, the side edges of the compression and the tension flanges are therefore assumed to have no chordwise displacement. Thus, the ends of each rib chord remain fixed in space during buckling. By the same reasoning each interior panel point of the rib truss should have no chordwise displacement if it is attached to the flanges. This attachment, however, might not be fully effective in the prevention of such displacements. Consequently, a calculation was made to determine the effect of chordwise displacements of the interior panel points of the rib truss. From the conditions of symmetry, it follows that the single interior panel point on the lower rib chord will have only a displacement normal to the flanges. The two interior panel points on the upper chord will have symmetrical displacements.

Let $-k_2$ and k_2 represent the ratio of chordwise to normal displacement of the first and second interior panel points, respectively, of the upper chord of the rib.

Let

$$\xi = \frac{Q_{\text{provided}}}{Q_{\text{required}}}$$

In figure 4, the ratio ξ is plotted against assumed values of k_2 for three conditions of the tension flange. The significant results obtained from figure 4 are given in table I. The value of k_2 that applies in each case is the value that makes ξ a minimum. Inspection of figure 4 or table I shows that $k_2 = 0$ gives values of ξ only slightly greater than the minimum values. It is therefore recommended that the effect of chordwise displacements be neglected in practical design calculations. Both figure 4 and table I show the very important effect of the tension flange in stabilizing the compression flange. This assumption greatly simplifies the calculation.

Some uncertainty exists with regard to assumptions made in the evaluation of GJ of the flanges. In order to study the importance of GJ, a special calculation was made to establish ξ when $GJ = 0$. This calculation gave $\xi = 1.11$ as compared with 1.18 in table I. These results show that GJ was not an important parameter in this particular problem.

From the definition of ξ, it follows that the ribs are adequately designed if ξ is equal to or greater than

unity. Consequently, the values in table I of ξ for the tension flange as constructed indicate a satisfactory design of the rib as a whole for the assumptions made. The dimensions of the rib truss are such that one of the assumptions made is open to question. This assumption will be discussed at some length in the CONCLUDING DISCUSSION.

DESIGN OF RIB-CHORD MEMBERS TO SPAN DISTANCE BETWEEN PANEL POINTS OF THE RIB TRUSS

Consideration is given only to the design of the rib-chord members adjacent to the compression flange of the box beam.

Theory.- The buckle pattern is assumed to be such that the compression flange will have no displacement along lines joining corresponding panel points in successive ribs. With this assumption, the theory of reference 1 applies provided that the restraint against rotation of the flange plate along these lines is either zero or infinite. In the design problem of reference 1, $f(y)$ was chosen to correspond to the condition of zero restraint. In this report the theory of reference 1 is extended to include any degree of elastic restraint. Let

$4S_o$ be the moment required to rotate the restraining members at the left end of the chord member through 1 radian

$4S_b$ the moment required to rotate the restraining members at the right end of the chord member through 1 radian

$$w = f(y) \sum_{m=1}^{m=\infty} b_m \sin \frac{m\pi x}{a} \tag{44}$$

The energy in the restraining members at both ends of all ribs is therefore

$$V = \frac{1}{2} \sum_i \left[4S_o \left(\frac{\partial w}{\partial y}\right)^2_{y=o} + 4S_b \left(\frac{\partial w}{\partial y}\right)^2_{y=b} \right] \tag{45}$$

or

$$V=2 \sum_i \left\{ S_o \left[f'(0)\right]^2 + S_b \left[f'(b)\right]^2 \right\} \left(\sum_{m=1}^{m=\infty} b_m \sin \frac{m\pi C_i}{a} \right)^2 \tag{46}$$

When this strain energy is included in the problem, the effect is to replace the γ of reference 1 by

$$\gamma + \frac{8b^2}{\pi^4 gD_x}\left\{S_o\left[f'(0)\right]^2 + S_b\left[f'(b)\right]^2\right\} \tag{47}$$

If the restraints are equal at each end of the rib-chord member, this expression becomes

$$\gamma + \frac{16b^2}{\pi^4 gD_x} S_o\left[f'(0)\right]^2 \tag{48}$$

In order to complete the solution, the function $f(y)$ must be derived. Consider the case of equal restraints. A sine curve will satisfy the condition of zero restraint at each end of the chord member. The deflection curve for a fixed-end beam under uniform load will satisfy the condition of infinite restraint. A combination of these two deflection curves can be made to satisfy any condition of restraint. It is therefore assumed that

$$f(y) = \sin\frac{\pi y}{b} + \epsilon\left[\frac{1}{2}\left(\frac{y}{b}\right)^4 + \frac{1}{2}\left(\frac{y}{b}\right)^2 - \left(\frac{y}{b}\right)^3\right] \tag{49}$$

where ϵ is a quantity that determines the relative contributions of the two deflection curves to $f(y)$. This form for $f(y)$ satisfies the conditions that $w = 0$ at $y = 0$ and $y = b$. Since $f(y)$ is symmetrical about $y = b/2$, the value of ϵ must be chosen to satisfy the following equation for equilibrium of moments at one end:

$$EI\left(\frac{\partial^2 w}{\partial y^2}\right)_{y=o} = 4S_o\left(\frac{\partial w}{\partial y}\right)_{y=o} \tag{50}$$

Substitution of $f(y)$ in equation (50) gives

$$\epsilon = \frac{\pi(4S_o)}{EI/b} \tag{51}$$

Thus for equal restraints at the ends of the rib-chord member, equations (7) and (8) of reference 1 in the notation of this paper become, respectively:

For the compression flange of finite length

$$\gamma = \frac{2/j}{\sum\limits_{k=o}^{k=\infty} \frac{1}{R_{2jk+q}} + \sum\limits_{k=o}^{k=\infty} \frac{1}{R_{2j(k+1)-q}}} - \frac{16S_c}{\pi^2 gD_x} \qquad (52)$$

For the compression flange of infinite length,

$$\gamma = \frac{\frac{1}{g}\left(\frac{b}{L}\right)^3}{\sum\limits_{k=o}^{k=\infty} \frac{1}{R_k{}'} + \sum\limits_{k=o}^{k=\infty} \frac{1}{R_k{}''}} - \frac{16S_c}{\pi^2 gD_x} \qquad (53)$$

In the definition of certain symbols the function $f(y)$ appears. The form of $f(y)$ as given by equation (49) should be used in all cases.

Design problem.- In order to complete the investigation of the ribs in the preceding design problem, the foregoing theory has been applied to a rib-chord member. The member chosen is the one between the two interior panel points of the upper rib chord. (See fig. 2.) This member has the same restraints at each end and, therefore, the foregoing theory applies.

An important factor that must be considered in the interpretation of the results is the effect of the attachment of the rib chord to the flange on the effective moment of inertia of the rib-chord member. In tables II and III and figures 5 and 6, which summarize the results, two assumptions are made. These assumptions are:

(a) The flange of the box beam does not influence the effective moment of inertia of the rib-chord member. (See fig. 3.)

(b) The flange of the box beam has such an influence as to lift the neutral axis of the rib chord up to the point of attachment to the flange, but only the area of the rib chord is effective in the computation of the moment of inertia. (See fig. 3.) Let ξ for the rib-chord member be defined as follows:

$$\xi = \frac{(EI)_{provided}}{(EI)_{required}} \qquad (54)$$

Thus ξ_a and ξ_b denote that $(EI)_{provided}$ is calculated in accordance with assumptions (a) and (b), respectively.

The panel points of the rib truss in figure 2 divide the compression flange into three equal parts. Thus the compression force in one part is $\frac{177670}{3}$ or 59,220 pounds. Similarly the width b is $\frac{76}{3} = 25.33$ inches. In tables II and III are given the significant results for these assumed values. Inspection of these tables shows the importance of assumptions (a) and (b). Hence, research is needed to establish the effective moment of inertia of the rib-chord member.

Tables II and III show the large effect of restraint at the ends of the rib-chord members and that the restraint "as provided" in this problem was so small as to have only a slight effect on the values of ξ. Comparison of corresponding values of ξ in tables II and III shows the important effect of GJ of the flange in the case of the rib-chord calculation.

The rib-chord member is adequately designed if ξ is equal to or greater than unity. Consequently, the values of ξ_a and ξ_b in tables II and III indicate that, for the restraint "as provided," the rib-chord members are not adequately designed if assumption (a) or assumption (b) is correct.

In order to establish the force in the compression flange for which the rib-chord member is adequate, figures 5 and 6 have been prepared. The broken-line curves in these figures for j infinite were included to show the small effect of increased length over that of five rib spaces. Figures 5 and 6 show that the allowable flange forces in a box beam of infinite length are, expressed as a percentage of the original assumed force of 59,220 pounds,

Assumption (a) and GJ as calculated	63.3
Assumption (b) and GJ as calculated	88.6
Assumption (a) and GJ = 0	57.0
Assumption (b) and GJ = 0	82.0

From this tabulation it is concluded that, if assumption (a) applies, the buckling load is between 57 and 63 per-

cent of the desired value of 59,220 pounds. If assumption (b) applies, the buckling load is between 82 and 89 percent of 59,220 pounds.

CONCLUDING DISCUSSION

In this paper the design of truss-type ribs for box beams has been resolved into two basic problems: (1) the design of the rib as a whole, and (2) the design of the chord members of the rib. Accordingly, this concluding discussion will be concerned with these two phases of the problem.

Design of rib as a whole.- In the design problem the rib had four web members. (See fig. 2.) For this rib it was assumed that

$$f_1(y) = f_2(y) = \sin \frac{\pi y}{b} \tag{55}$$

Had there been many web members, it would have been justifiable to have assumed a form for the functions $f_1(y)$ and $f_2(y)$ that took into account the effect of fixity at each end of the rib as a whole. Had this different form been chosen, the calculated values of ξ would have been higher than the values given in table I. It is therefore concluded that, for a given amount of material, a rib with many web members is desirable. Other important factors will, of course, limit the number of web members.

In the calculation of the rib as a whole, it is necessary to assume that the chord members are adequately designed. If they are inadequately designed, the rib as a whole cannot function in a completely satisfactory manner.

The function of the rib-chord members adjacent to the compression flange is to stabilize the compression flange between the panel points of the rib truss. The function of the rib-chord members adjacent to the tension flange is to gather up the stabilizing forces from the tension flange and to transmit these forces to the panel points of the rib truss. When $f_1(y)$ was assumed to have the value given by equation (55), it was assumed that the rib-chord member adjacent to the tension flange was properly performing

this function. Now, the results obtained in the design problem showed that the tension flange plays a very important part in stabilizing the compression flange. Consequently, the rib-chord members adjacent to the tension flange must be very important members in the rib. These chord members are much longer than the rib-chord members adjacent to the compression flange, which were found to be seriously undersize. Although no calculation has been made to establish the required size of the rib-chord members adjacent to the tension flange, there is reason to suppose that these members may also be undersize.

If the rib-chord members adjacent to the tension flange are not adequately designed, the tension flange does not provide the stabilizing influence to the compression flange that was inherently assumed in the design problem. It can therefore be concluded that the actual values of ξ are less than the numerical values of table I. Hence, the rib as a whole is probably less adequately designed than was calculated.

<u>Design of rib-chord members</u>.- In the design problem, a restraint was calculated for the rib-chord members and this restraint was inserted through ϵ in the function $f(y)$, which was also assumed to apply in the deflection of the flange between ribs. It was further assumed that the flange had no normal displacement along lines joining corresponding panel points in successive ribs. Both of these assumptions introduce restraints not actually present in the structure; consequently, the calculated values of ξ are higher than they would have been if more accurate assumptions had been made. This fact indicates that the rib-chord members should be strengthened more than indicated by the calculations of the design problem. It also means that the allowable load for the sizes provided is less than calculated.

After the establishment of the fact that the rib-chord members are not adequately designed to stabilize the compression flange, the next logical question is how the design should be corrected. From the conservative solution of reference 1, the required EI of a rib-chord member is concluded to vary as the fourth power of the width b. This conclusion suggests that the best method of strengthening the rib chords is to reduce the distance between the panel points of the rib truss. Consequently, additional web members in the rib truss are desirable. These added web members also strengthen the rib as a whole.

In order for the rib-chord members and the rib as a whole properly to perform their function of stabilizing the compression flange, the connection of the web members to the chord members of the rib must be properly designed. If the web and the chord members of the rib are of adequate size and the connection is sufficient to develop the full strength of these members, the entire rib is adequately proportioned. As herein used the term "full strength" includes the axial, the bending, and the shear strength of the members. If the restraint of the web members is to be relied upon in the design of the rib-chord members, it is necessary that the joints of the rib truss be properly designed to perform this function; otherwise, a failure of the joints can be expected.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 7, 1942.

APPENDIX

RESULTS OF COMPRESSION TESTS ON CORRUGATED PANELS FIVE BAYS LONG LATERALLY SUPPORTED BY A SIMPLIFIED RIB STRUCTURE

After the foregoing theory and examples had been completed, a number of test specimens of the type shown in figure 7 were constructed in order to provide experimental data on the design of rib-chord members to span the distance between panel points of the rib truss. The proportions of the specimens were so chosen as to show the effect of variation in distance between panel points of the rib truss on the strength of the compression flange of the box beam. The width of these specimens were as follows:

Number of specimens provided	Width of specimen (in.)	Distance between center lines of supporting tubes (in.)
2	19.3	18.6
1	24.6	23.9
1	30.0	29.3
2	35.4	34.5

The specimens were mounted for test in the 1,200,000-pound-capacity testing machine in the NACA structures research laboratory, as shown in figure 8. The simplified rib structure used in the test specimens was attached to I-beam supports. On the assumption that the chord member of the rib structure possessed a bending stiffness sufficient to make it equivalent to a rigid member between its end supports, the proportions of the I-beams were made such that their stiffness, in combination with that of the supporting tubes, was greater than the minimum required to be equivalent to a rigid support. The ratios of the stiffness provided to this minimum required stiffness for the different panel widths are:

Panel width (in.)	Ratio
19.3	Greater than 2.50
24.6	Greater than 2.02
30.0	Greater than 1.71
35.4	Greater than 1.50

The minimum required stiffness used in the computation of these ratios was computed by use of equation (72) of reference 2.

The foregoing ratios were calculated on the assumption that the ends of the panel were supported on knife edges, whereas actually the panels were flat-ended. Because the support provided the test panel was greater than the minimum required to be equivalent to a <u>rigid</u> support and because no structure can provide greater than rigid support, the load developed in the test panel should be equal to or greater than the load that would be developed if the panel were part of an airplane structure.

For the first series of tests, which included one panel of each width, the length of the tube supports C (see fig. 7) was 32.5 inches. In these tests, ultimate failure occurred in the supporting tubes; the type of failure is evident in figure 8. In the case of the specimen 30.0 inches wide, a failure (fig. 9) also developed in the chord member of the rib structure. It is not known whether this failure occurred before or after the supporting tubes failed, as shown in figure 10. In this panel, as well as in the widest panel, there was a marked tendency for the chord member of the rib structure to deflect

normal to the panel prior to failure. Only the 30.0-inch-wide panel appeared to be damaged by these tests. The 35.4-inch-wide panel, which appeared to be undamaged, was equipped with a new set of 32.5-inch tube supports and retested. This panel developed about the same load as in the first test.

The panels 19.3, 24.6, and 35.4 inches wide were then retested with the length of the tube supports reduced to 12.5 inches. In these tests, failure occurred in the chord member of the rib structure rather than in the supporting tubes. (See figs. 11, 12, and 13.)

The results of the tests are plotted in figure 14 and indicate clearly how the ultimate compressive load on the panel per inch of width is reduced as the panel width is increased. It can also be seen that the shortening of the tube supports did not increase the ultimate loads. Either the first tests damaged the panels to such an extent that no higher loads could be developed, or the shortening of the tubes did not raise the strength of the combination. The fact that the failure seemed ~~to be~~ concentrated in the tubes when they were long and shifted to the chord member of the rib structure when the tubes were shortened indicates that the proportions of the specimen in the first case were very nearly such as to make the two types of failure equally likely. Shortening of the tubes would then alter the proportions in a manner that would bring about the transition from failure of the tubes to failure of the chord members. The fact that a failure did occur in the chord member for one of the specimens tested with the longer tubes further indicates that the ultimate loads for the two types of failure were very nearly equal. On the other hand, the fact that the panels 19.3 and 24.6 inches wide with 12.5-inch tube supports failed by a small margin to develop the loads achieved by these same panels with 32.5-inch tube supports indicates that the panels may have been damaged somewhat by the first tests. The duplicate specimens 19.3 and 35.4 inches wide, which have not been tested, may be used in a further study of the effect of the size of the tube supports, should it appear advisable to continue this investigation.

The significant conclusion to be drawn from the test results presented in this appendix is that the failure was not restricted to the panel between ribs, as is customarily assumed in design, but rather involved the panel and rib structure as a unit and, in every case, caused failure in the rib structure.

If the type of failure that occurs in the structure differs from the type that is assumed in the design calculations and if the strength for the type of failure assumed in the design calculations is correctly computed, the structure must have developed a strength less than that computed in design. This principle is fundamental in structural theory and is verified by the results of these tests. The pin-end column strength for a panel 16 inches long and 25.33 inches wide is 59,220 pounds according to the example in the body of this report. As the failure in these tests was not of this type, but involved a failure of the rib structure, a lower strength should be expected. From figure 14, the experimental strength of a panel 25.33 inches between tube supports is estimated to be 1980 pounds per inch width of panel. The total load for a 25.33-inch width is therefore 50,150 pounds. This value represents 85 percent of the pin-end column strength of the panel between ribs.

The experimental value plotted in figures 5 and 6 at an abscissa of $\xi = 1$ and an ordinate of 85 percent shows that assumption (b) is approximately correct for the evaluation of the effective EI for the rib chord in this particular example. Had the proportions of the rib chord differed in relation to the proportions of the corrugations, this assumption might not have been so well checked by experiment.

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2. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc., 1936.

TABLE I

RESULTS FOR RIB AS A WHOLE

[j = 5; L = 16 in.; GJ as calculated]

Condition of tension flange	ξ (for k_2 = 0)	ξ (for k_2 at minimum)
Absolutely rigid	1.66	1.66
As constructed	1.18	1.17
No rigidity	.58	.55

TABLE II

RESULTS FOR RIB-CHORD MEMBER

[j = 5; L = 16 in.; GJ as calculated]

Restraint at ends of rib-chord member	ϵ	ξ_a	ξ_b
Infinite	∞	1.509	3.596
As provided	1.50	.280	.667
Zero	0	.265	.632

TABLE III

RESULTS FOR RIB-CHORD MEMBER

[j = 5; L = 16; GJ = 0]

Restraint at ends of rib-chord member	ϵ	ξ_a	ξ_b
Infinite	∞	0.965	2.298
As provided	1.50	.191	.454
Zero	0	.183	.436

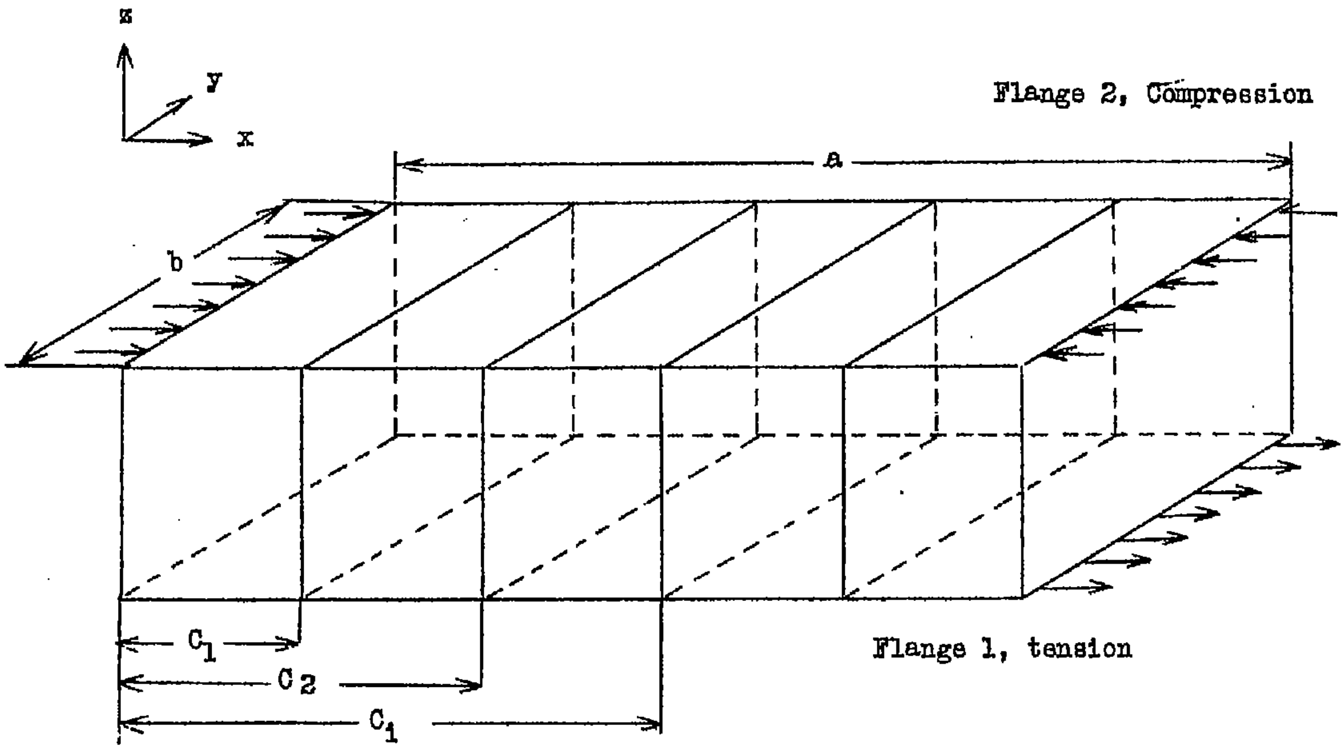

Figure 1. - Box beam under pure bending

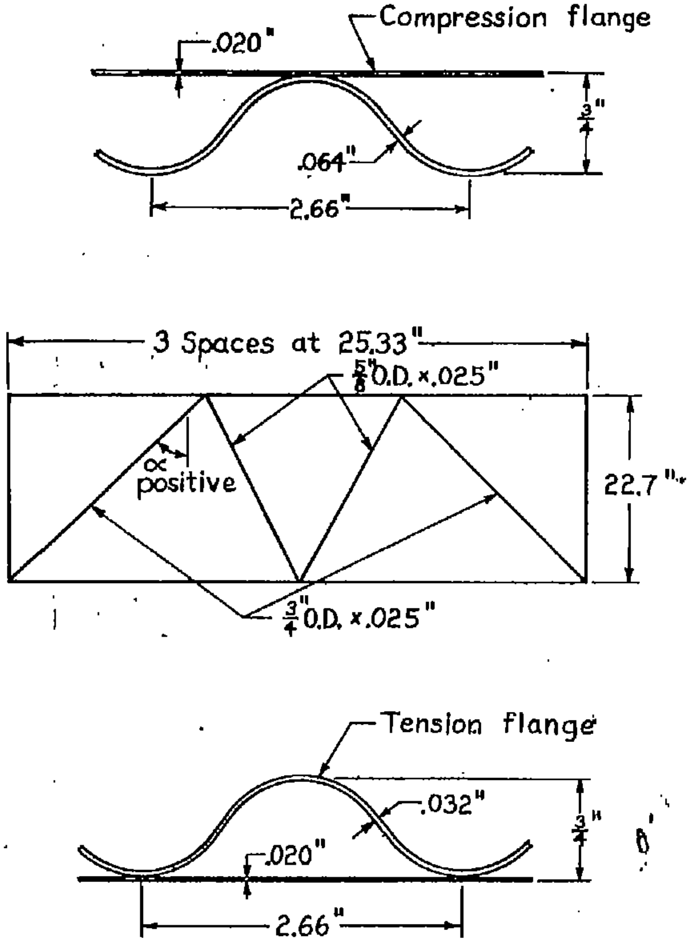

Figure 2.- Rib and box-beam flanges for design problem. Flanges riveted to rib chords at each corrugation.

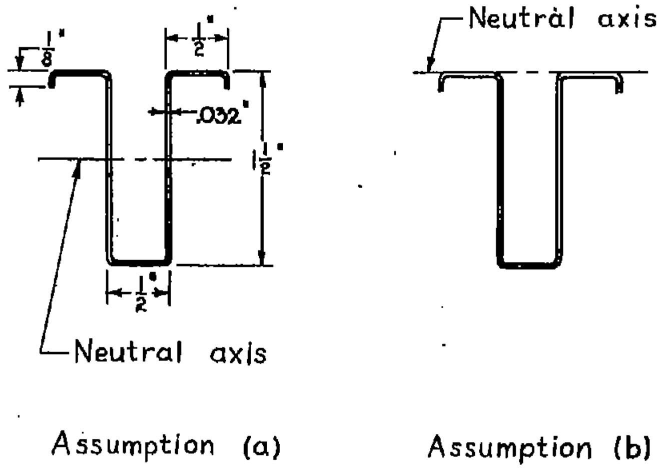

Figure 3.- Cross section of rib chord members for design problem.

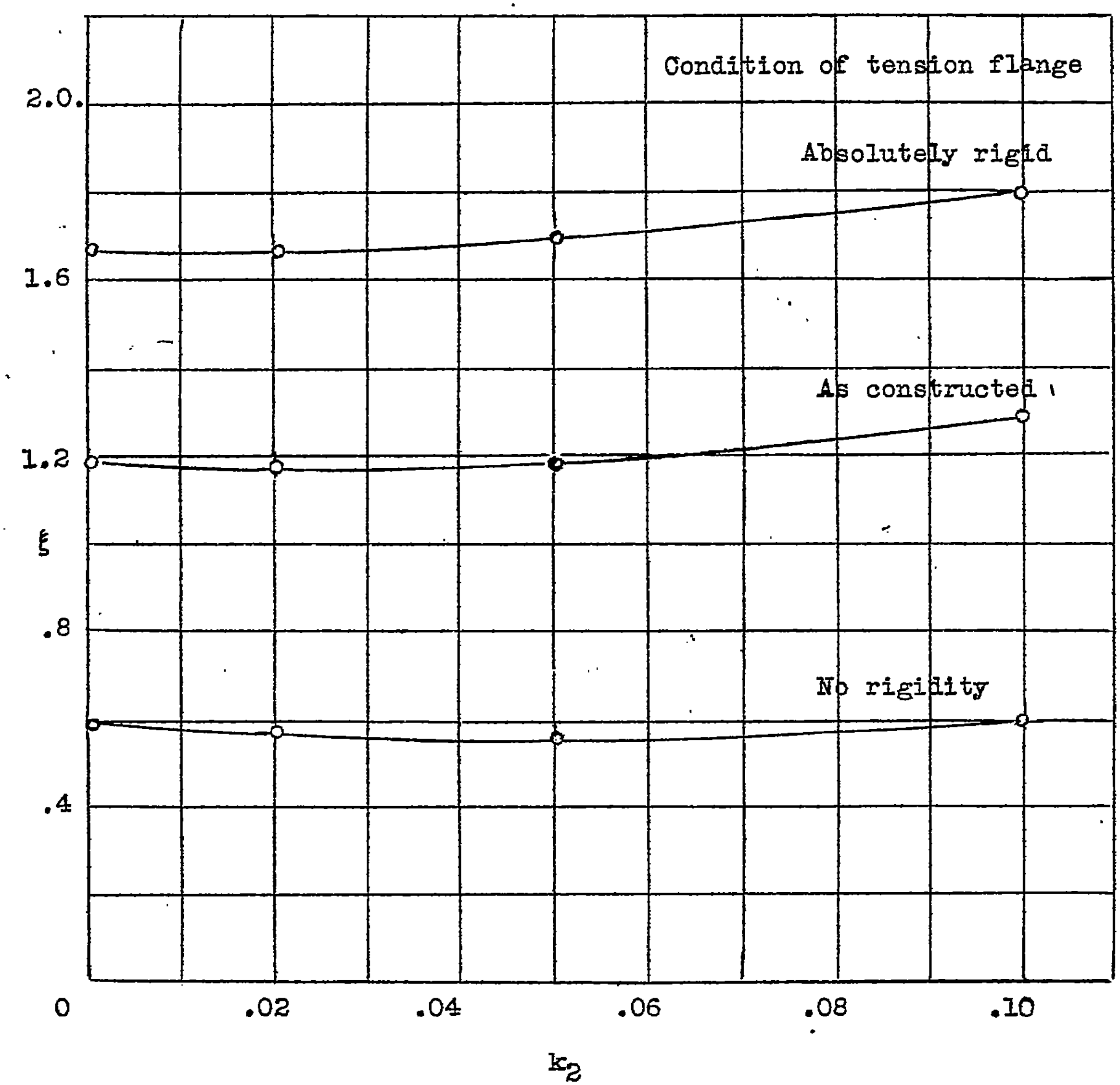

Figure 4. - Variation of ξ with k_2.

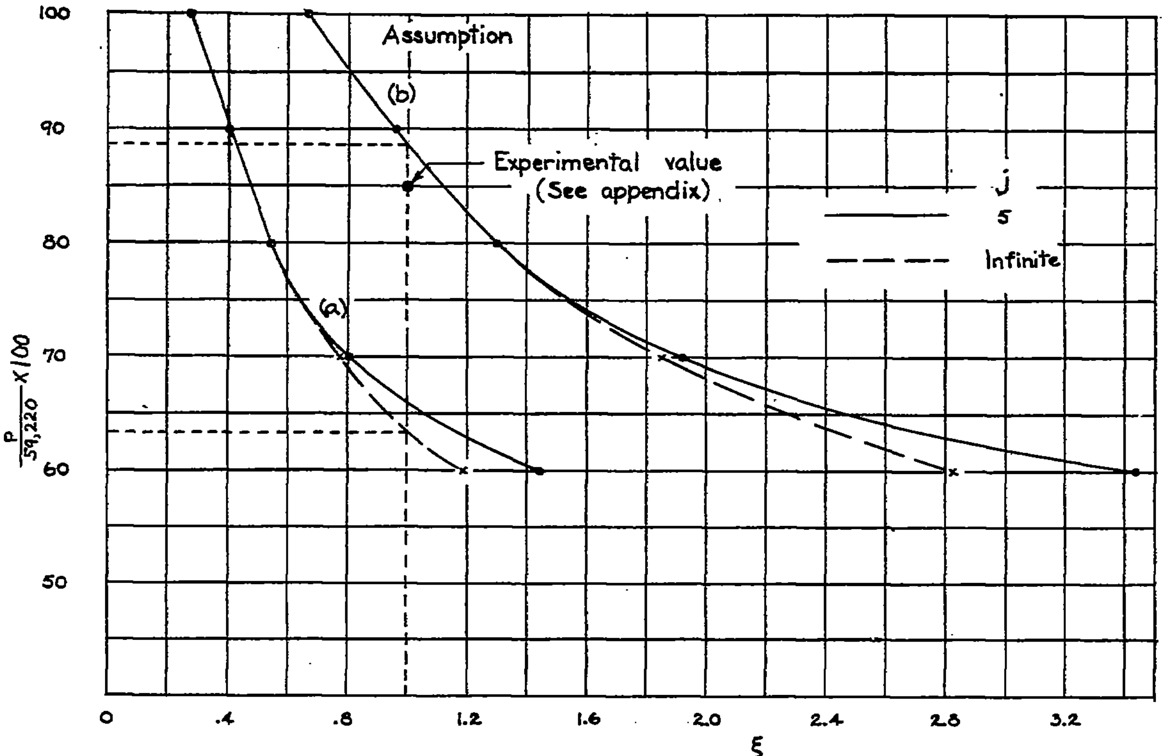

Figure 5.- Results for rib-chord member (GJ as calculated) L, 16 inches.

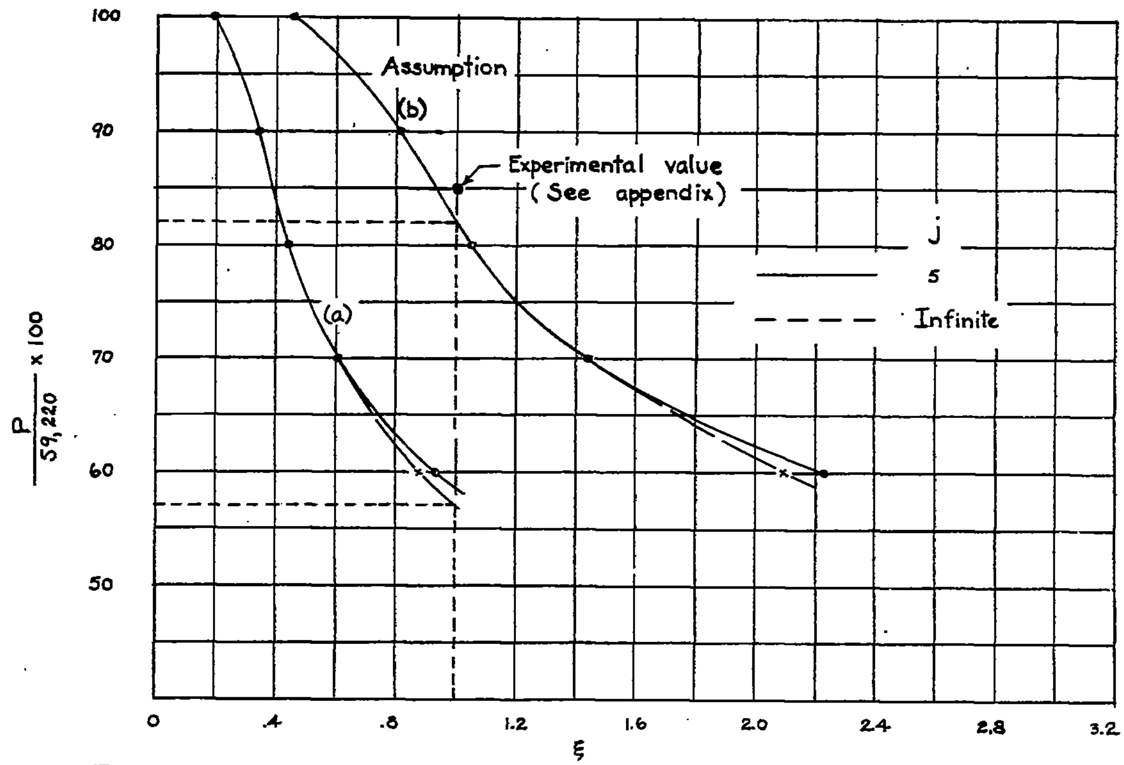

Figure 6.- Results for rib-chord member (GJ = 0) L, 16 inches.

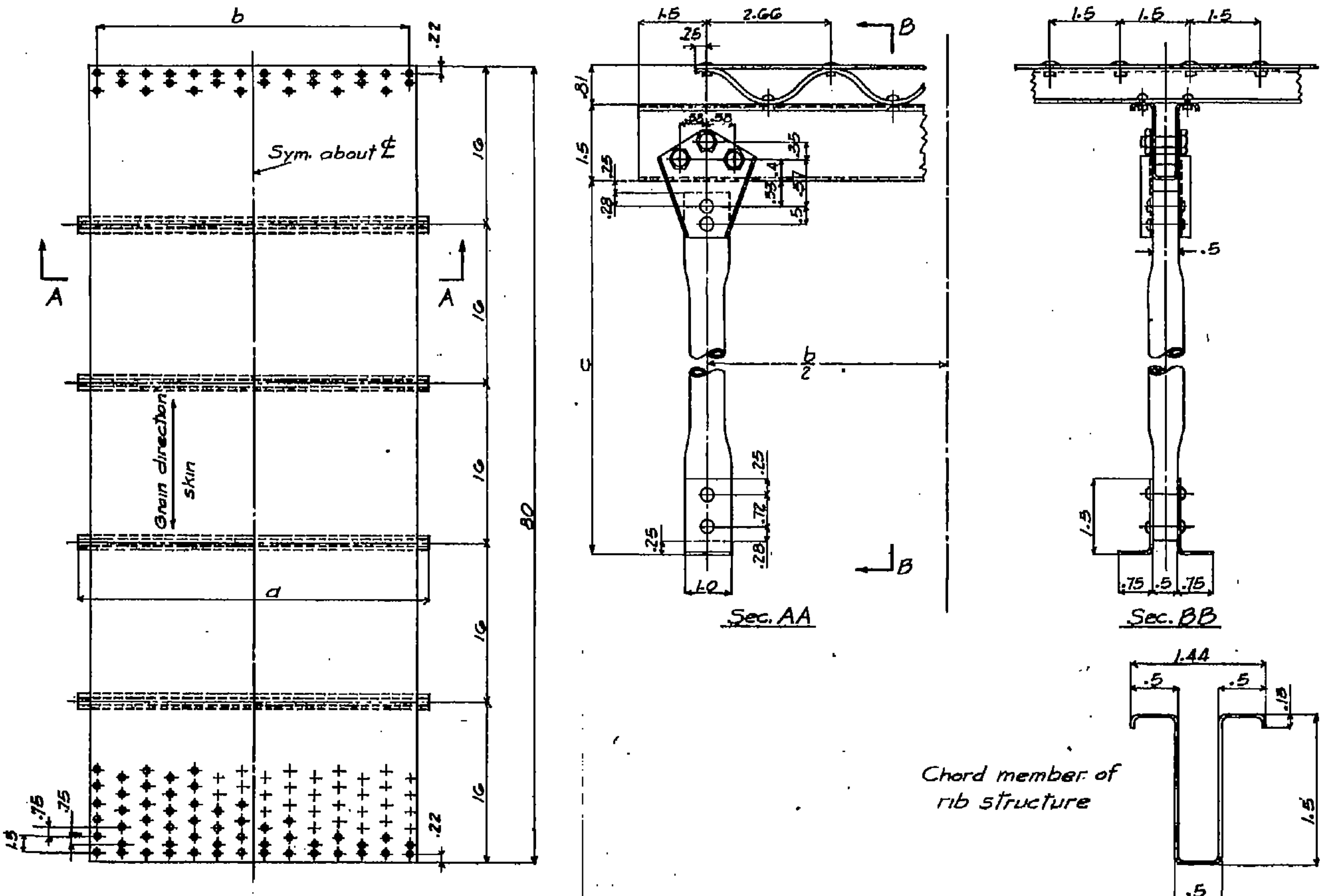

Figure 7.- Test specimen.

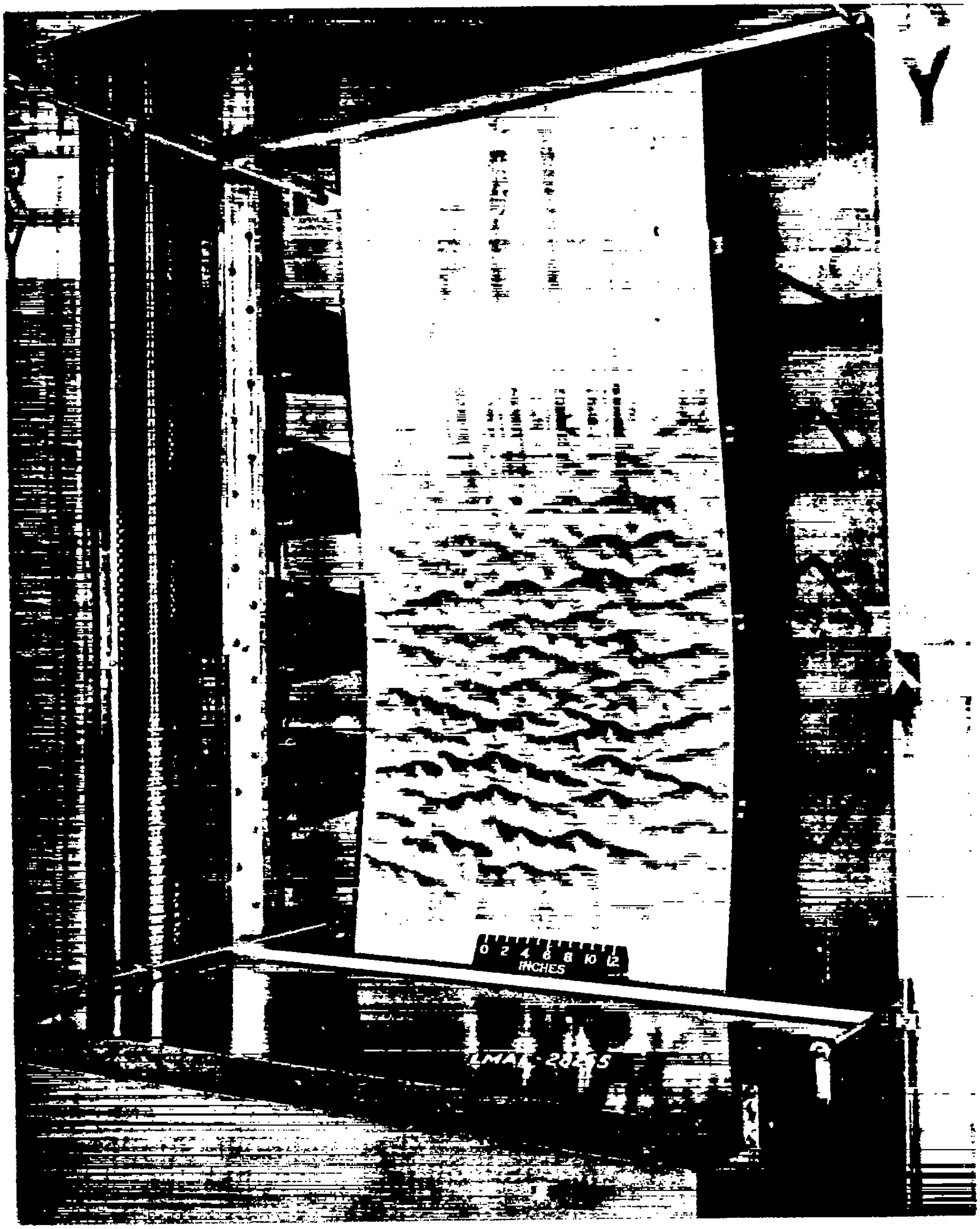

Figure 8. - Panel 35.4 inches wide after failure with supporting tubes 32.5 long.

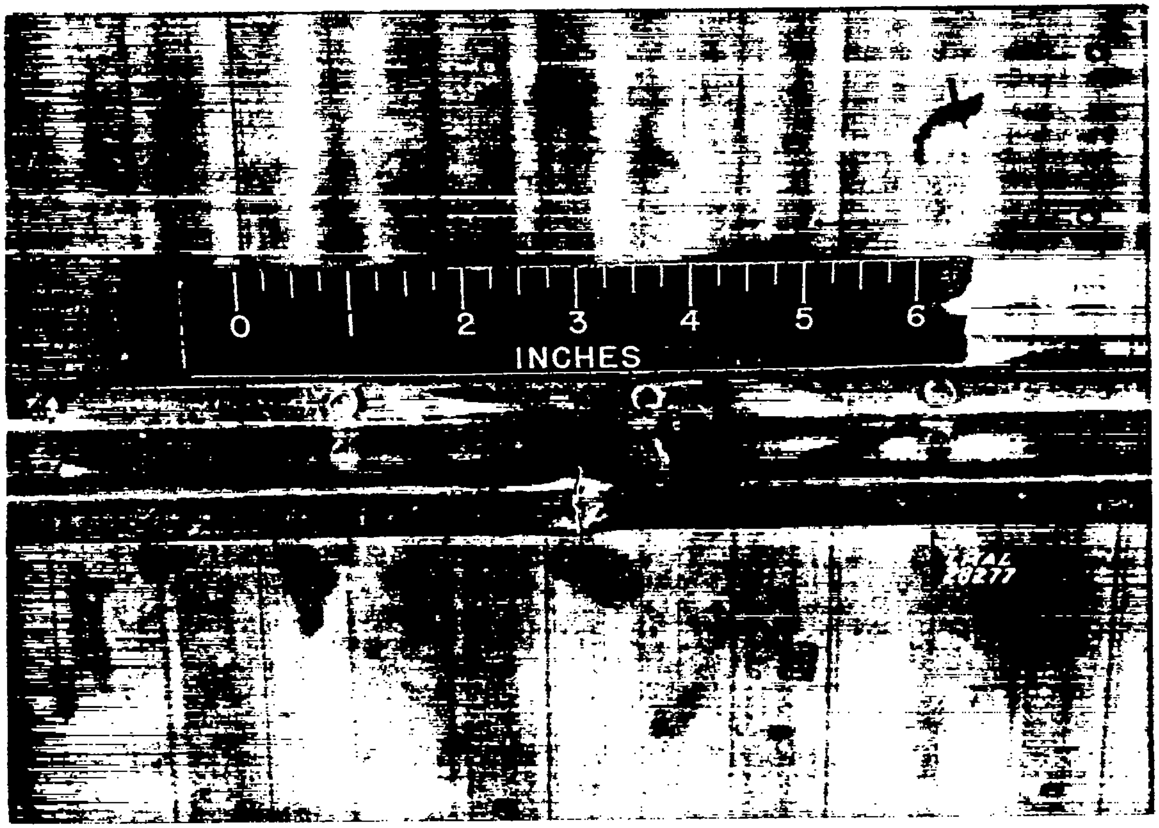

Figure 9.- Failure of the chord member of the rib structure of panel 30.0 inches wide with supporting tubes 32.5 inches long.

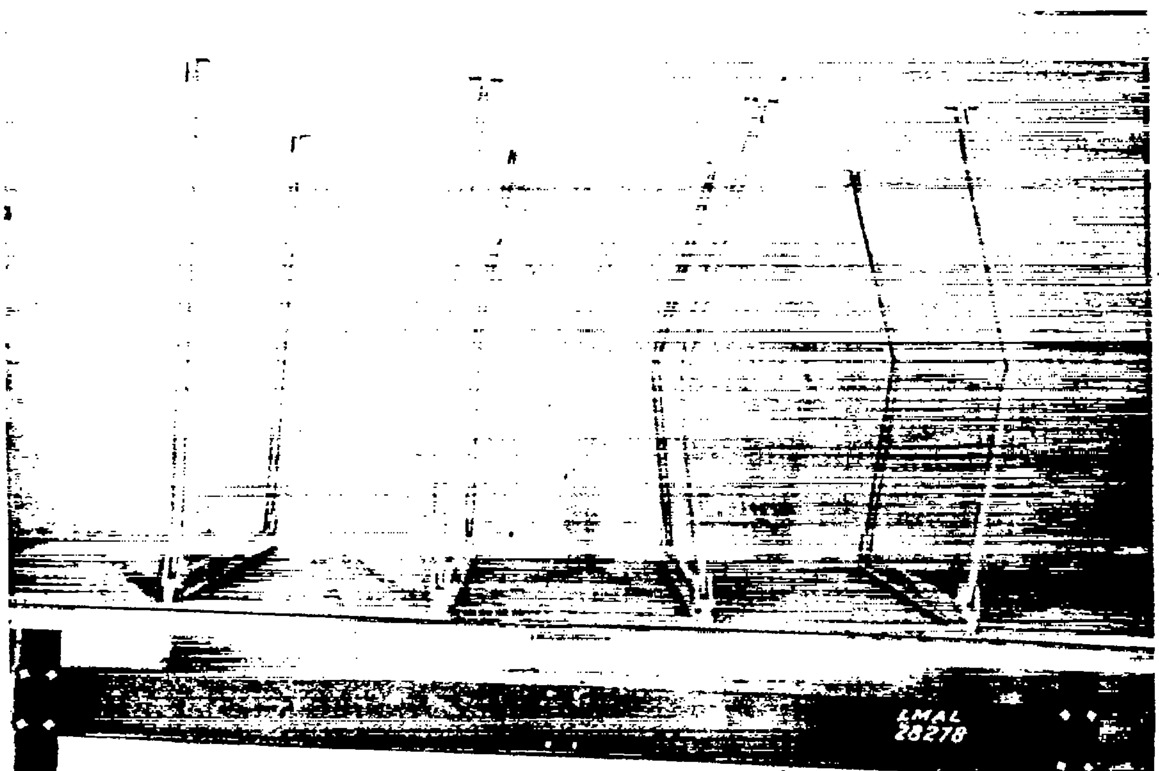

Figure 10.- Panel 30.0 inches wide after failure with supporting tubes 32.5" long.

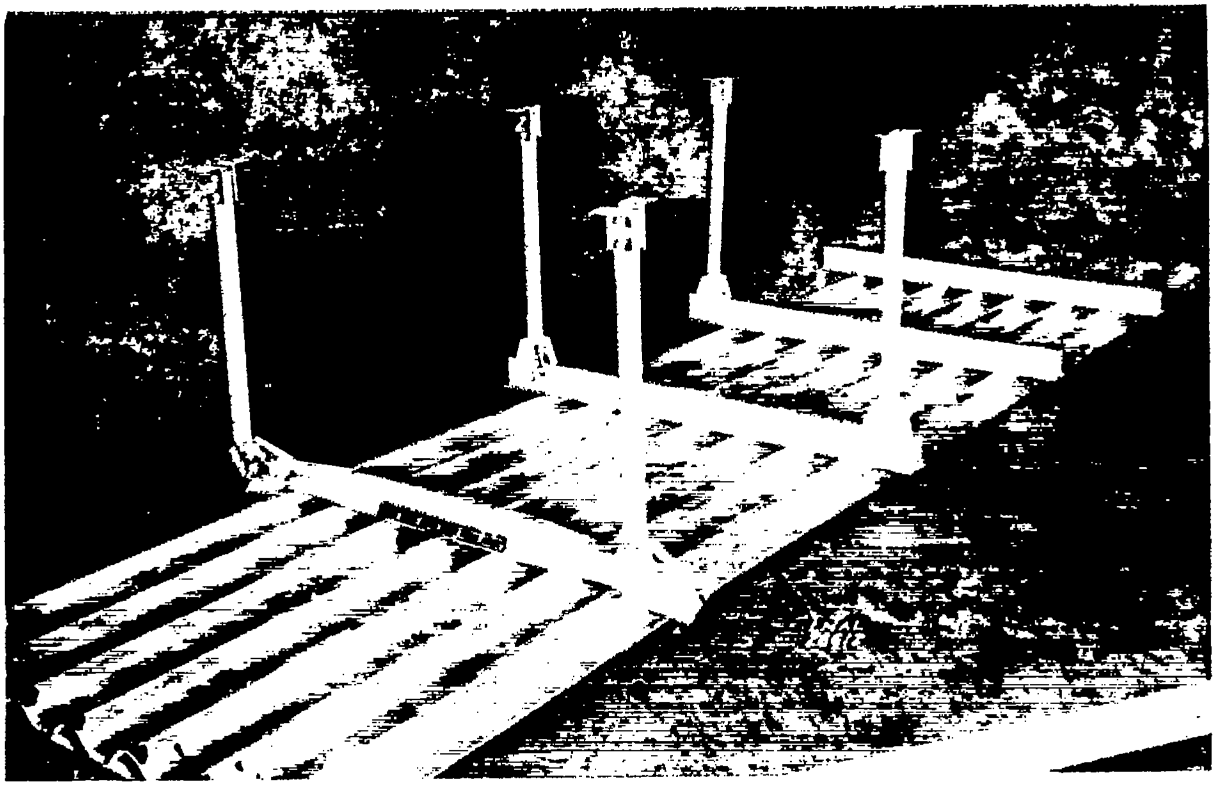

Figure 11.- Panel 19.3 inches wide after failure with supporting tubes 12.5 inches long.

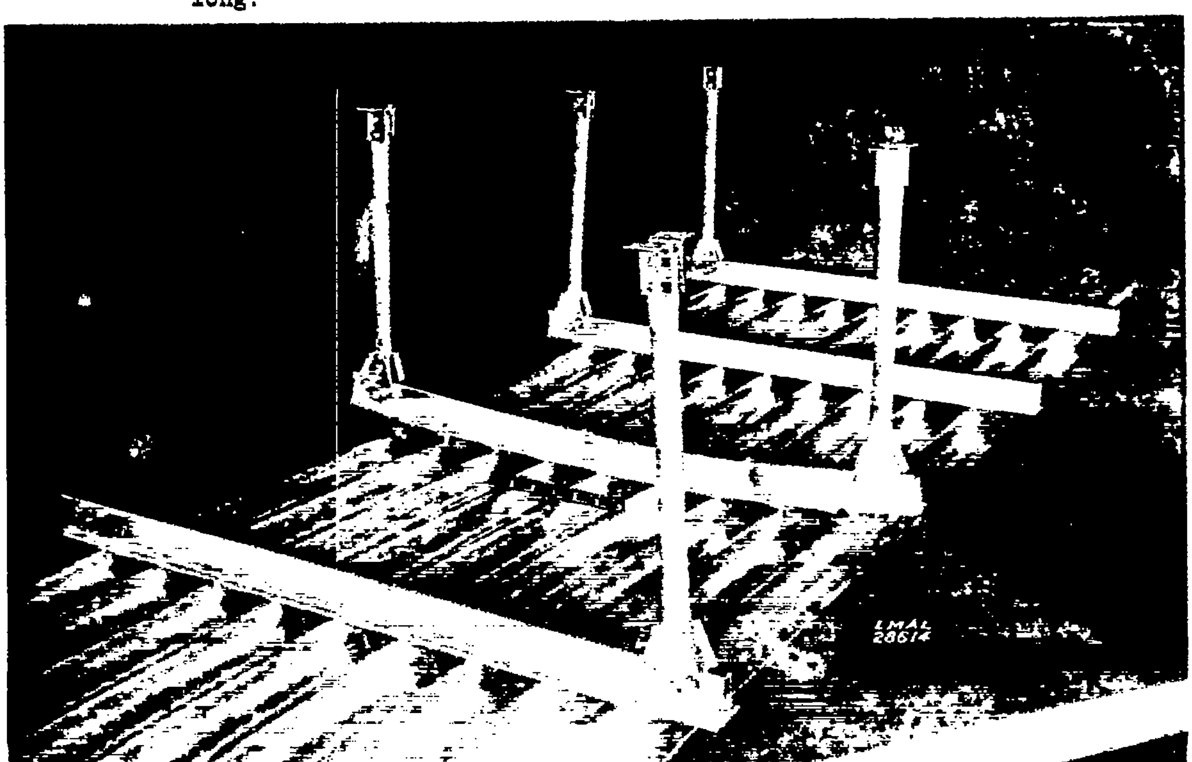

Figure 12.- Panel 24.6 inches wide after failure with supporting tubes 12.5 inches long.

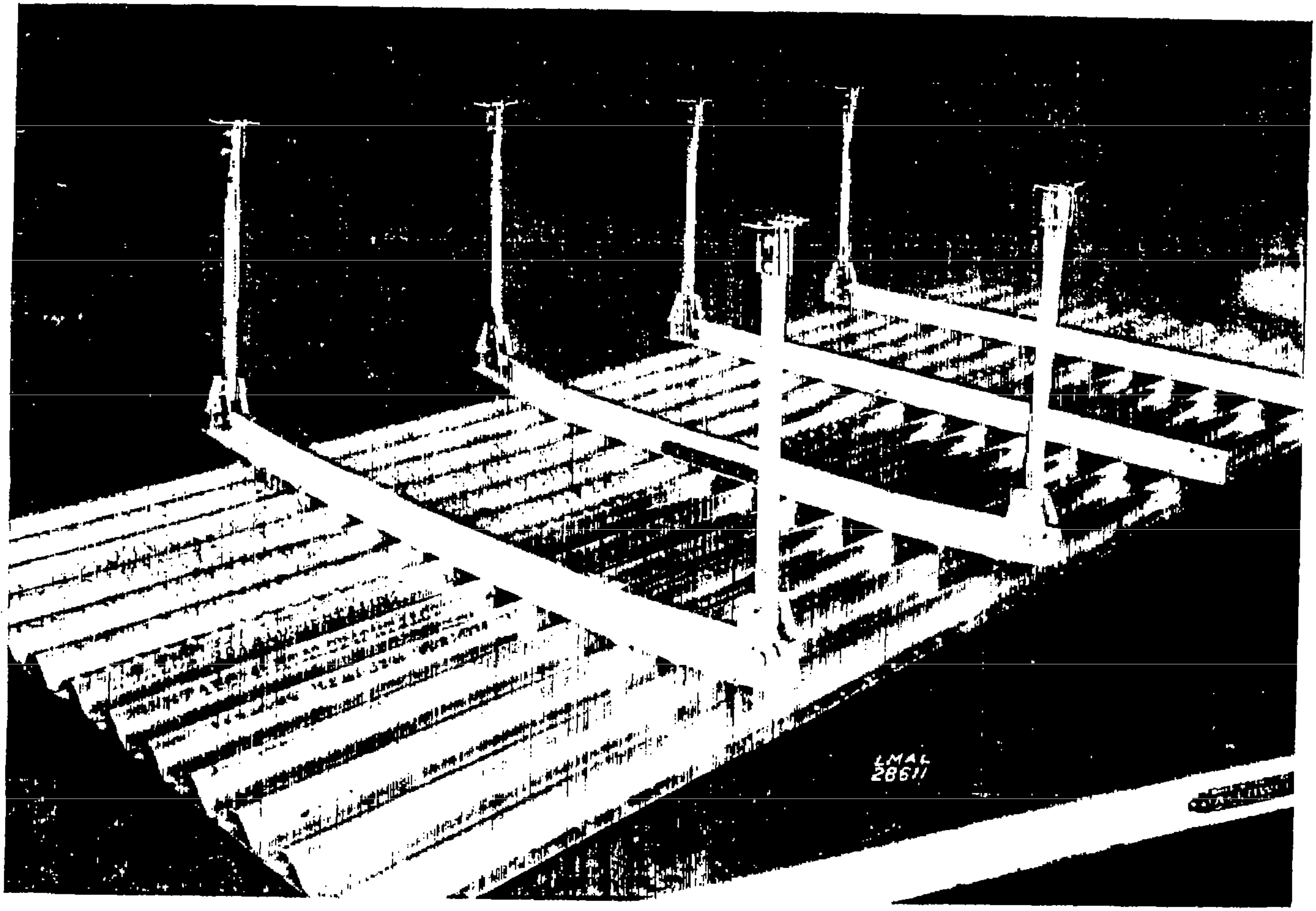

Figure 13.- Panel 35.4 inches wide after failure with supporting tubes 12.5 inches long.

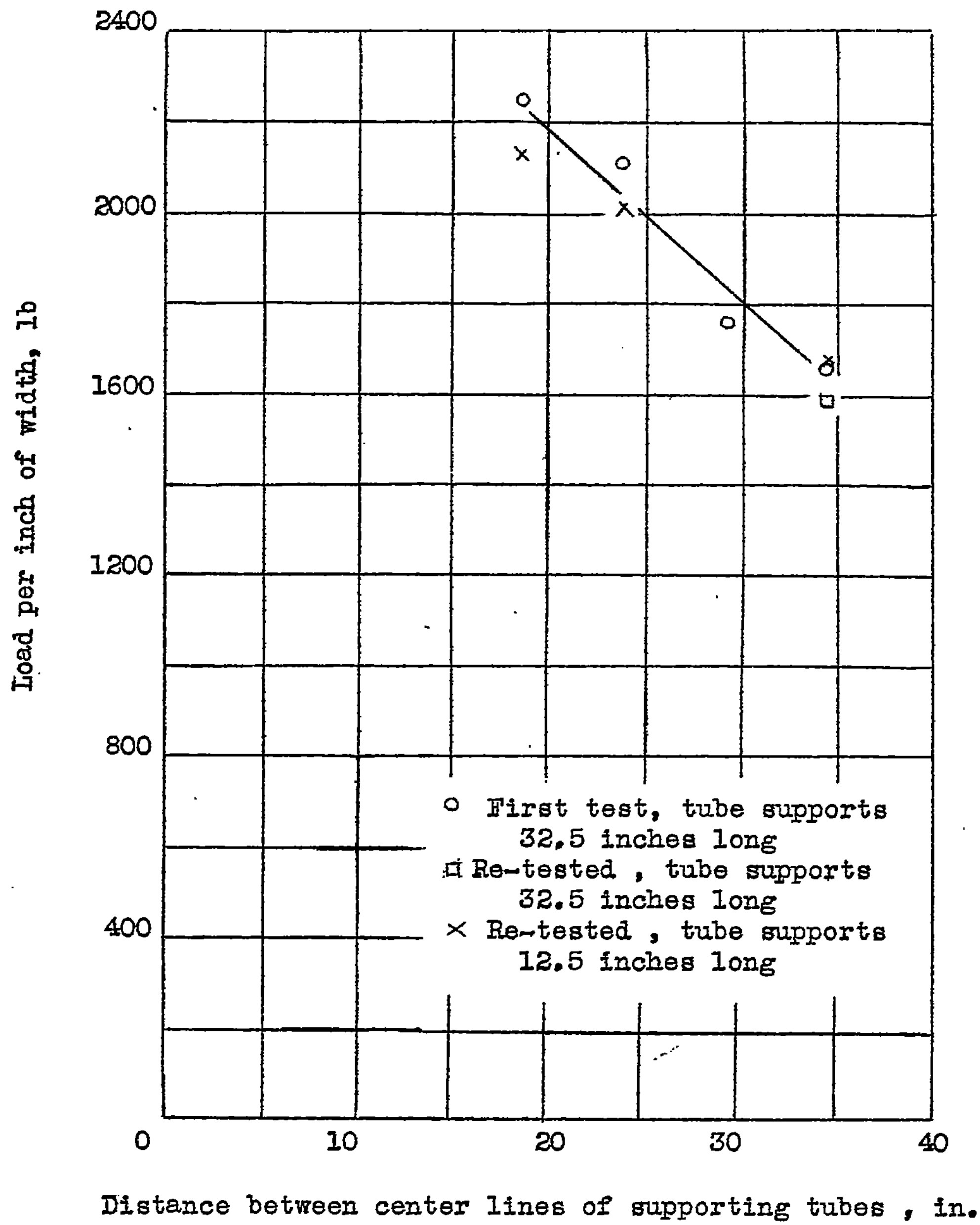

Figure 14.- Effect of panel width on compressive strength.